Material Balance Calculations Using the Excel Spreadsheet

Introduction

Material balance is a fundamental petroleum reservoir engineering tool that can be used to provide an understanding of a reservoir and the influence of any connecting aquifer. The basic requirements for the application of material balance to a reservoir include:

1) The hydrocarbon and water production from all wells producing from a reservoir must be summed to provide a total reservoir production history.

2) Some knowledge of the average reservoir pressure history must be known. Utilizing the average pressure history of the reservoir suggests that material balance is best applied in reservoirs where relatively small pressure gradients exist.

3) PVT properties must be expressed with sufficient accuracy using a so-called "Black Oil" model. Solubility of gas in oil and oil formation volume factor must be expressed as a simple function of pressure. Gas is assumed to be insoluble in water. This requirement typically eliminates volatile oil and rich gas condensate reservoirs from consideration for material balance calculations.

The ability to express the pressure and production history in this so-called "tank" model often allows an accurate estimation of initial hydrocarbons in place and/or the productivity and size of any connecting aquifer. Material balance can often be applied fairly early in the life of reservoir before it is fully delineated by drilling which can aid in selecting new well locations and spacing once the drive mechanism is understood.

The use of material balance by practicing reservoir engineers has recently fallen from favor with the development of reservoir simulation. This loss of knowledge and experience is unfortunate when the simplicity of material balance calculations is considered compared to reservoir simulation. Indeed, all the data required for a material balance analysis is collected for any reservoir simulation study. It is the experience of this reservoir engineer that if material balance is successful in characterizing the aquifer and initial hydrocarbons in place, that the time required for the history matching process in a reservoir simulation project may be shortened by an order of magnitude! Furthermore, if material balance is unsuccessful in those reservoirs that can be described by a tank model, problems with the data are indicated and any reservoir simulation is doomed to failure!

While various commercial material balance programs exist, the purpose of this endeavor was to provide an easy to use "freeware" program that can be used in the relatively simple reservoirs that comprise the vast majority of material balance studies. Furthermore, this work was targeted toward those problems where available pressure data is sparse. This is accomplished by using all production data available along with assumed values of initial hydrocarbons in place and aquifer properties to calculate a pressure history for comparison to any historic measured values of pressure. This is in contrast to the method of using the measured pressure data in the calculations to estimate water influx in a stair-step fashion usually utilized in a method commonly referred to as an "XY" plot. This commonly used XY plot can provide excellent results when the input data includes regularly recorded bottom-hole pressures over relatively short time intervals. The accuracy of the XY plot method must be called into question, however, as the time between measured pressure values increases and the pressure data becomes more sporadic.

For example, consider a problem that has a measured pressure after two years of production. Assume that the reservoir was produced for the first year at 10,000 BOPD and 30,000 BOPD/day the second year. In the common XY plot method, the water influx calculations would be identical if instead the reservoir had been continuously produced at 20,000 BOPD for two years, *or even if it had been produced at 40,000 BOPD for the first year and then shut-in for the second!* Clearly the approximations required for water influx calculations can yield erroneous results as the pressure data becomes sparse and the production rates become erratic.

A very powerful feature of this program is that in addition to initial hydrocarbon in place and aquifer properties, other parameters that may not be known with confidence can also be varied. For example, the initial pressure of the reservoir may not be exactly known and is extremely important in the calculations. Formation compressibility may not be known and a value for it can be determined that yields a best fit. Any combination of parameters can be fixed or varied. A particularly useful application fixes the initial hydrocarbons in place at the volumetric value determined from the initialization of a simulation model to allow the determination of appropriate aquifer properties for the model.

Material Balance Concept

The development of the general oil material balance equation can be found in any petroleum reservoir engineering textbook including <u>Applied Petroleum Reservoir Engineering</u> (B. C. Craft and M. F. Hawkins, revised by R. E. Terry) and <u>Fundamentals of Reservoir Engineering</u> (L. P. Dake). This equation simply states that as the pressure in the reservoir falls, the oil, gas and water must be allowed to expand. The volume of this expansion in reservoir barrels, along with a reduction in pore volume and any fluid injection, must be equal to the total fluid production also expressed in reservoir barrels. Although not its most simple algebraic form, the oil GMBE can be written as follows allowing each term in the equation can be readily identified:

$$\underbrace{N\,(B_t - B_{ti})}_{\textit{Oil Expansion}} + \underbrace{N\,m\,B_{ti}\,\frac{(B_g - B_{gi})}{B_{gi}}}_{\textit{Gas Cap Expansion}} + \underbrace{(N\,B_{ti} + N\,m\,B_{ti})\,c_w\ \Delta p\ S_w / (1-S_w)}_{\textit{Water Expansion}}$$

$$+ \underbrace{c_f\ \Delta p\,(N\,B_{ti} + N\,m\,B_{ti}) / (1-S_w)}_{\textit{Formation Contraction}} + \underbrace{W_e}_{\textit{Water Influx}} + \underbrace{W_I\,B_{wI}}_{\textit{Water Injection}} + \underbrace{G_I\,B_{gI}}_{\textit{Gas Injection}}$$

$$= \textit{Oil \& Disolved Gas Production} + \textit{Free Gas Production} + \textit{Water Production}$$

$$= \ N_p\,B_t + N_p\,(R_p - R_{soi})\,B_g + W_p B_w$$

Similarly, the gas GMBE can also be written in a form that allows each term expressed in reservoir barrels to be readily identified:

$$\underbrace{G\,(B_g - B_{gi})}_{\textit{Gas Expansion}} + \underbrace{G\,B_{gi}\ c_w\ \Delta p\ S_w / (1-S_w)}_{\textit{Water Expansion}} + \underbrace{W_e}_{\textit{Water Influx}} + \underbrace{c_f\ \Delta p\ G\,B_{gi} / (1-S_w)}_{\textit{Formation Expansion}}$$

$$= \ G_p\,B_g + \ W_p B_w$$

$$= \textit{Gas Production} + \textit{Water Production}$$

While the current reservoir pressure in the above equations is only explicit in the "Δp" found in the formation and water expansion terms, it is also required to determine the current formation volume factors and cumulative water influx volume. Determining the reservoir pressure at each time step therefore requires an iterative calculation. This program utilizes the "Newton-Rhapson" technique to determine the current reservoir pressure at each time step for a given production history, aquifer size, aquifer productivity, and initial hydrocarbon volume. The calculated pressure history can then be compared to the actual measured pressure history and the aquifer properties or initial hydrocarbon volume adjusted until an acceptable history match is obtained.

The previously described procedure of manually adjusting aquifer properties and hydrocarbon volume until a history match is obtained can actually be automated through the clever use of the built-in Excel™ "Solver" add-in. Solver can, for instance, be used to minimize the sum of the square of the vertical distances between calculated and measured pressures. It is tempting to allow Excel to perform the history matching process without any manual attempts. While this author almost always ultimately allows Solver to arrive at the best history match, some effort is first spent manually attempting to determine the match. Although arguably this seems like an inefficient use of time, it is primarily done for three reasons: 1) bad data points can usually be readily identified and excluded from the ultimate Solver solution, 2) Solver can be unstable and providing it with starting values that are close to the ultimate best fit values can improve stability and decrease calculation time, and 3) perhaps most importantly, manual manipulation of the problem can provide an important insight into the sensitivity of the model to changes in the input parameters providing a level of confidence in the uniqueness of the history match.

Water Influx

Much has been written regarding water influx and the subject is thoroughly discussed in the previously mentioned reservoir engineering texts. This program utilizes the "Fetkovich" analytical aquifer model that approximates the unsteady-state aquifer model of Hurst and van Everdingen. This model was chosen for two reasons. First, the required calculations are simple and straightforward. Secondly, the Fetkovich model can be directly input into many modern reservoir simulators, including Eclipse™. While it would have been possible to use the more rigorous Hurst and van Everdingen aquifer model, the improvement in accuracy was not believed to be worth the additional computational time and programming effort. These seems particularly true when the inaccuracy of water production measurements is considered and the fact that the Hurst and van Everdingen model itself makes simplifying assumptions, including the assumption that aquifer productivity is constant when it most certainly falls as water encroaches into the reservoir.

Fetkovich defined a term known as the maximum encroachable water, which is the amount of water that an aquifer would supply if the pressure were dropped to zero. It is simply the product of aquifer pore volume, initial pressure, and total aquifer compressibility.

$$W_{ei} = \underbrace{V_{aq}}_{\text{Aquifer Pore Volume}} \times \underbrace{P_i}_{\text{Initial Pressure}} \times \underbrace{c_{aq}}_{\text{Aquifer Compressibility}}$$

Assuming constant aquifer compressibility, the average aquifer pressure at any point in time can be simply calculated based on the fraction of the total encroachable water that has encroached from the aquifer.

$$P_{aq} = P_i \times (1 - \underbrace{W_e}_{\text{Cumulative Water Influx}} / W_{ei})$$

Fetkovich used the term aquifer productivity index that can be used to estimate the instantaneous water influx rate. While knowledge of the aquifer geometry, dimensions, permeability, and water viscosity can be used to calculate this term

$$q_{aq} = \underbrace{J_{aq}}_{\text{Aquifer Productivity Index}} \times (P_{aq} - \underbrace{P_{res}}_{\text{Reservoir Pressure}}) = \frac{dW_e}{dt}$$

(see Dake page 328 for example calculations), *typically less is known about the aquifer than the reservoir being studied*. For this reason, the aquifer productivity index and the encroachable water term discussed previously are used in this program as history matching values to describe the aquifer.

Fetkovich algebraically manipulated and integrated these equations to arrive at an expression describing the water influx for a constant pressure drop and a specified time. Theoretically, this equation requires tedious superposition in a manner similar to that required by the Hurst and van Everdingen method. Fetkovich's most significant contribution was to determine that his solution could be applied in a stepwise fashion using the equation to the right. This eliminated the need for superposition while providing approximate results of acceptable accuracy.

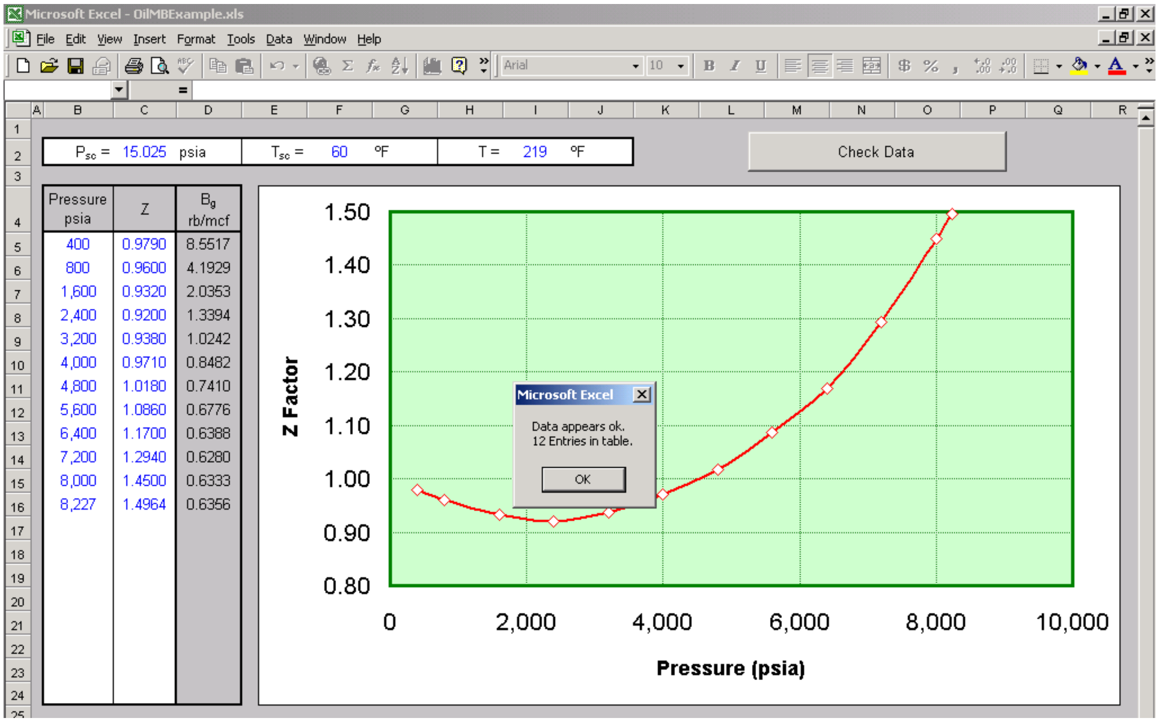
$$\Delta W_{en} = \frac{W_{ei}}{p_i} \left(\overset{\textbf{Aquifer Pressure At Beginning of Time Step}}{p_{n-1}} - \underset{\textbf{Reservoir Pressure At End of Time Step}}{p_{Rn}} \right) \left(1 - e^{-J\, p_i \Delta t_n / W_{ei}}\right)$$

It is not the intention of this narrative to replace the complete discussion of the Fetkovich aquifer found in most reservoir engineering texts. This discussion is intended to show that through the use of the Fetkovich aquifer, only two parameters are required to describe an aquifer and estimate water influx, the aquifer productivity index (J) and the aquifer maximum encroachable water (W_{ei}). As mentioned previously, less is typically known about the aquifer than the reservoir being studied. For that reason, it is recommended that these two parameters be varied in the history matching process. Once final results are obtained, the indicated values can be verified as reasonable.

Data Input

Two blank spreadsheets are available for material balance analysis, LSUOilMB.xls for oil reservoir analysis and LSUGasMB.xls for gas reservoir analysis. Before using the blank spreadsheets, it would be best to examine two completed problems, OilMBExample.xls and GasMBExample.xls. The individual worksheets within the workbook are arranged in the order that they would typically be completed. Data entry in the grey colored areas of the worksheets is prevented using Excel's protection feature allowing data entry into only those cells with a white background. The protection feature does not prevent adjusting the graphs that often require manual adjustment of the axis scales to obtain the desired presentation. There may be situations that require protection to be removed and this can be accomplished by either turning the protection off for an individual worksheet or turning off global protection (see Excel help).

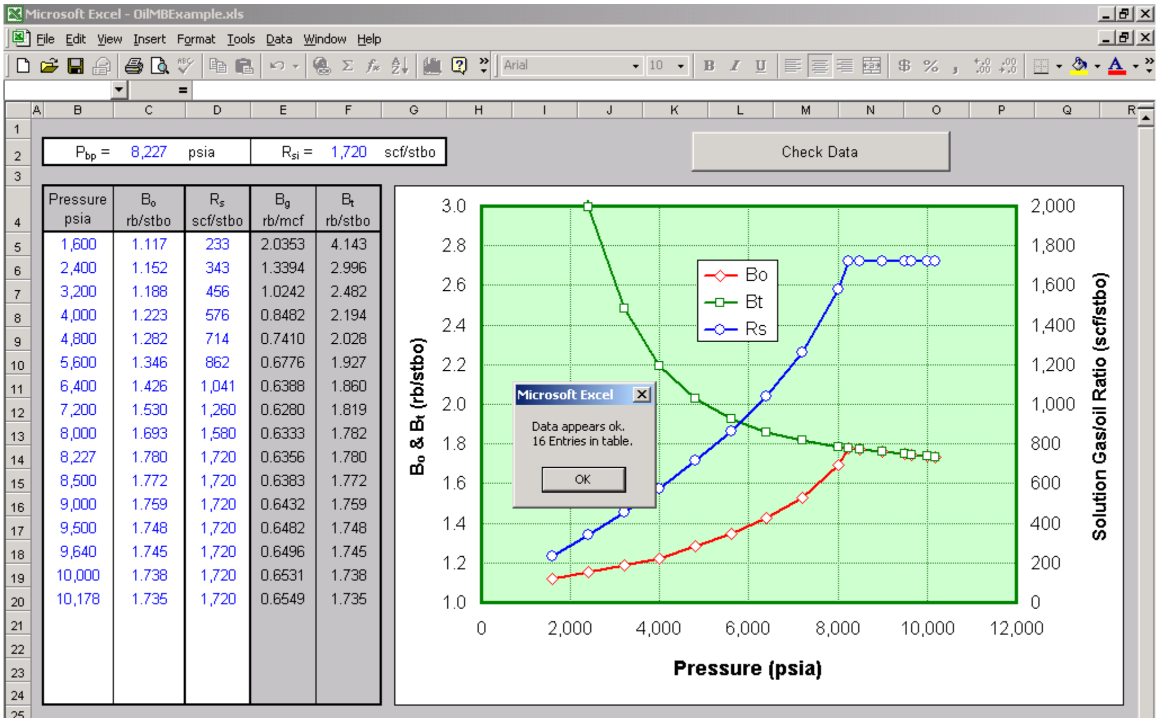
The required data input include the gas "Z" factors and standard conditions, oil PVT properties (for oil problems), production and pressure history, formation compressibility, water compressibility, water saturation, and initial guesses for the initial hydrocarbons in place and aquifer properties. The first worksheet, tabbed "Z Factors", from the example oil material balance problem is shown below:



It is important that the pressure values be entered in ascending order, monotonically increasing, as shown in the example. A total of twenty values may be entered and a rather complicated interpolation scheme is used to obtain values between the entered data points. The pressure range should encompass all expected pressure values. If any pressures are encountered outside the entered range, *the program will continue by linearly extrapolating using the last two data points in the table.* Note that a Visual Basic button labeled "Check Data" can be pressed that performs a rudimentary check of the entered data. This check is not foolproof, however, so care should be exercised when entering all data.

For oil problems, the next set of data that would normally be entered is the oil PVT properties. The worksheet labeled "Oil PVT" from the example oil problem follows. Again, required data is in the portions of the sheet that are not colored grey and pressure values should be monotonically increasing as shown in the example. It is very important that the correct bubble point pressure and initial solution gas-oil ratio be entered both at the top of the sheet and within the pressure table. For undersaturated oil problems that never fall below the bubble point, it should be possible to begin the table with the bubble point, and for initially saturated problems with an initial gas cap, the table can be ended with the bubble point pressure. Again, rudimentary checks of the data are made with the "Check Data" button.

Pressure and production data, initial guesses at hydrocarbons in place and aquifer properties, water and formation compressibility, and water saturation are entered on the



Pressure psia	B_o rb/stbo	R_s scf/stbo	B_g rb/mcf	B_t rb/stbo
1,600	1.117	233	2.0353	4.143
2,400	1.152	343	1.3394	2.996
3,200	1.188	456	1.0242	2.482
4,000	1.223	576	0.8482	2.194
4,800	1.282	714	0.7410	2.028
5,600	1.346	862	0.6776	1.927
6,400	1.426	1,041	0.6388	1.860
7,200	1.530	1,260	0.6280	1.819
8,000	1.693	1,580	0.6333	1.782
8,227	1.780	1,720	0.6356	1.780
8,500	1.772	1,720	0.6383	1.772
9,000	1.759	1,720	0.6432	1.759
9,500	1.748	1,720	0.6482	1.748
9,640	1.745	1,720	0.6496	1.745
10,000	1.738	1,720	0.6531	1.738
10,178	1.735	1,720	0.6549	1.735

sheet tabbed "Calculations." Note that the column for water production can be viewed as a "net" value and can be negative in situations where water injection is present. Considering the inaccuracies of water production measurements, water formation volume factor was assumed to equal unity. If it is desired to include B_w, adjust the input water production to reservoir barrels prior to inputting the values to the spreadsheet.

Initial guesses for initial hydrocarbon in place should obviously be larger than cumulative production. Initial guesses at aquifer properties can vary widely. It is often useful to assume an infinite acting aquifer by entering an extremely large value for W_{ei}, say 1E9 reservoir barrels, and a limited value for productivity index, perhaps on the order of 1 BWPD/psi. Manual attempts at history matching the aquifer properties usually show that early time data is influenced strongly by aquifer productivity and later time pressure is more strongly influenced by aquifer size.

The value of "m", the ratio of gas cap volume to oil volume, should be set to zero for initially undersaturated oil problems. The formation and water compressibility can be often be neglected by entering values of zero without a significant loss in accuracy if any free gas is initially present in the system. Pressure, cumulative time, and cumulative volumes are typically entered monthly, but can actually be entered at any time interval desired.

Once the required data is entered, the formulae found in cells G15 through L15 must be copied down for each of the entered time periods. Note that cell L15 may appear blank but does in fact contain a formula. Copying these formulas should be saved for the last step because once copied, recalculation of the spreadsheet can take several seconds with each change. The completed “Calculations” sheet should appear similar to the following example.

Microsoft Excel - OilMBExample.xls

N =	13.7	MMSTBO
m =	0.0	
J =	1.00	BPD/psi
W_{ei} =	66.8	MMBbls
P_i =	10,180	psia
S_{wi} =	0.200	fraction
C_w =	2.5E-06	psi^{-1}
C_f =	3.0E-05	psi^{-1}

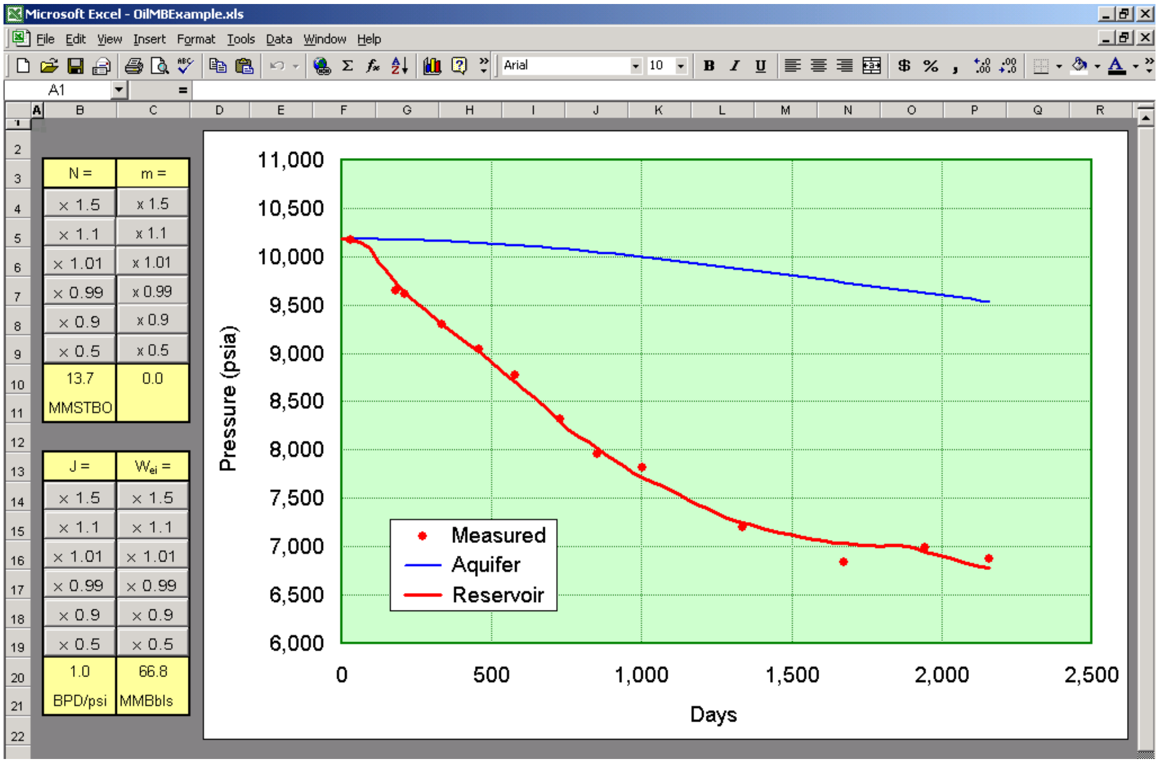
Calculated		
Original:		
HCPV =	23.72	MMBbls
PV =	29.66	MMBbls
Oil PV =	23.72	MMBbls
Gas PV =	0.00	MMBbls
B_{ti} =	1.735	rb/STBO
B_{gi} =	0.6549	rb/MCF
C_{factor} =	3.813E-05	psi^{-1}
Sum Sq's =	7.99E+04	

Days	N_p STBO	G_p MCF	W_p STBW	Meas P psia	Rp SCF/STBO	P psia	We RB	P_{aq} psia	Q_{aq} RB/day	Delta P^2/1E6
0	0	0	0			10,180	0	10,180	0	
30	4,000	3,000	0	10,178	750	10,176	57	10,180	4	3.29E+00
61	28,000	21,000	0		750	10,153	526	10,180	26	
92	100,000	56,000	0		560	10,096	2,227	10,180	83	
120	186,000	229,000	0		1,231	9,955	6,514	10,179	223	
151	285,000	338,000	0		1,186	9,848	15,058	10,178	328	
181	374,000	486,000	0	9,643	1,299	9,728	26,673	10,176	446	7.19E+03
212	444,000	612,000	0	9,618	1,378	9,634	41,918	10,174	537	2.67E+02
242	513,000	733,000	0		1,429	9,546	59,318	10,171	623	
273	582,000	851,000	0		1,462	9,461	79,876	10,168	704	
304	652,000	970,000	0		1,488	9,377	102,936	10,164	784	
334	722,000	1,080,000	0	9,293	1,496	9,299	127,561	10,161	858	3.65E+01
365	791,000	1,192,000	0		1,507	9,223	155,252	10,156	929	
395	857,000	1,298,000	0		1,515	9,154	184,091	10,152	994	
426	924,000	1,396,000	0		1,511	9,090	215,801	10,147	1,052	
457	995,000	1,515,000	0	9,040	1,523	9,015	249,497	10,142	1,122	6.19E+02
485	1,059,000	1,623,000	0		1,533	8,949	281,759	10,137	1,183	

The final worksheet displays the calculated pressure profile compared to the measured pressures. The graphical result for the completed history match for the example problem follows. Note that the values of the measured pressures closely match the calculated pressure profile. Inspection of the worksheet reveals several visual basic buttons to the left of the graph. These buttons can be very handy for manually attempting to obtain a history match. Each click of the button manually changes the indicated parameter by the fraction indicated. Manipulation in this fashion quickly gives a new calculated pressure profile for comparison to the measured values. It is important to note, however, that the user is not limited to changing only the indicated parameters. For example, if initial pressure or formation compressibility is unknown, these parameters can also be easily varied in the original “Calculations” worksheet. The user will note that calculation time slows with changes to this worksheet compared to the graphical display due to multiple screen refreshes that occur on the “Calculations” sheet.

Automatic History Matching

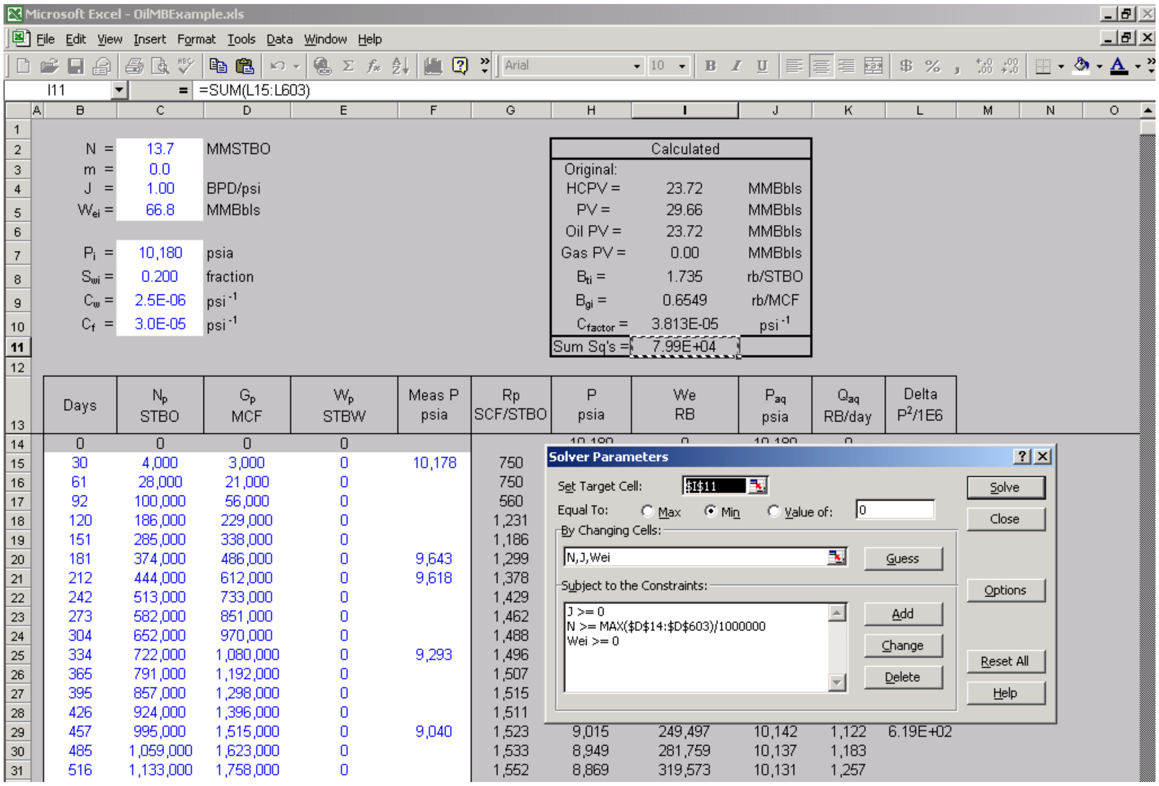
Column “L” in the Calculations sheet calculates the square of the vertical distance between the calculated and measured pressure values whenever a measured pressure value is present. (Actually, the distance squared divided by 1E6 to make the value more manageable.) These values are then summed in cell I11 on the Calculations worksheet. One measure of a best fit of the measured data is to minimize the sum of these squares, a



so-called "least squares" fit of the data. While minimizing this value can be accomplished manually, Excel has the ability to accomplish the operation automatically. The simplest method is a utility known as "Goal Seek" that can be found under the Tools menu. Goal seek instructs Excel to "find a specific result for a cell by adjusting the value of one other cell". For example, we could instruct Excel to attempt to make the cell containing the sum of the squares to zero by changing the original oil in place.

A much more powerful option within Excel is an add-in named "Solver". Solver far exceeds the power of Goal Seek in that numerous parameters can be changed while honoring specified constraints. Solver provides an incredible power to Excel that can replace tedious computer programming for a wide variety of iterative engineering calculations. It is not installed in the standard installation of Excel but must be installed for future use by selecting the Tools menu, then Add-Ins, and then checking the box for the Solver Add-in. Note that the original Excel installation disk may be requested if Solver has never been used in a particular installation. Once initially installed, it will be available under the Tools menu. Note that to use Solver on a worksheet, the sheet must be unprotected (select Tools, Protection, and Unprotect Sheet).

An example of the Solver add-in follows. In this example, Solver has been instructed to minimize the sum of the squares by changing the initial oil in place (N), the aquifer productivity index (J), and the maximum encroachable water (W_{ei}). Notice also that constraints have been put on the parameters, namely that the aquifer productivity index and maximum encroachable water must be greater than or equal to zero, and the initial oil in place must be greater than the cumulative oil production.



Depending upon the complexity of the problem, Solver can require a fairly significant amount of computer time for its calculations. Its progress can be observed in the lower left hand corner of the spreadsheet. By default, it will pause after 100 seconds and prompt whether to continue, and it will also pause after 100 iterations. These defaults can be changed under the "options" button if desired, along with such things as convergence criteria and the calculation method employed. Most problems typically converge in around five minutes with less than ten iterations. It may be wise to change the maximum iterations from 100 to perhaps 20 since the process is likely unstable if convergence is not rapidly achieved and 100 iterations could take an hour or more to complete.

As previously mentioned, Solver can become unstable and it is best to start the calculations with values at least close to the final solution. It is highly recommended that the spreadsheet be saved prior to the use of Solver. The Newton-Rhapson method employed in the material balance program usually converges in less than five iterations. It also can occasionally become unstable and will halt after 100 iterations and display a message that it is experiencing convergence problems. Beginning the calculations with values nearer the final solution usually eliminates these convergence problems.

Nomenclature

B_g – gas formation volume factor, reservoir barrels/MSCF
B_{gi} – initial gas formation volume factor, reservoir barrels/MSCF
B_{gI} – injected gas formation volume factor, reservoir barrels/MSCF
B_t – two phase oil formation volume factor, reservoir barrels/stock tank barrel
B_{ti} – initial two phase oil formation volume factor, reservoir barrels/stock tank barrel
B_w – water formation volume factor, reservoir barrels/stock tank barrel
B_{wI} – injected water formation volume factor, reservoir barrels/stock tank barrel
C_{aq} – aquifer total compressibility, $C_f + C_w$, psi $^{-1}$
C_f – pore volume compressibility, psi $^{-1}$
C_w – water compressibility, psi $^{-1}$
G – initial free gas in place, MSCF
G_p – cumulative gas production, MSCF
G_I – cumulative gas injection, MSCF
J_{aq} – aquifer productivity index, reservoir barrels / day - psi
m – initial gas cap reservoir volume / initial oil reservoir volume
N – initial oil in place, stock tank barrels
N_p – cumulative oil production, stock tank barrels
P_{aq} – aquifer pressure, psia
P_i – initial reservoir pressure, psia
P_{n-1} – aquifer pressure at beginning of time step “n”, psia
P_{res} – reservoir pressure, psia
P_{Rn} – reservoir pressure at end of time step “n”, psia
q_{aq} – aquifer influx rate, reservoir barrels / day
R_p – cumulative gas production / cumulative oil production, SCF/stbo
R_{soi} – initial solution gas-oil ratio, SCF/stbo
S_w – water saturation
V_{aq} – aquifer pore volume, reservoir barrels
W_e – cumulative water influx, reservoir barrels
W_{ei} – maximum encroachable water volume, reservoir barrels
W_I – cumulative water injection, stock tank barrels
W_p – cumulative water production, stock tank barrels
Δp – initial pressure less current pressure, psi
Δt_n – length of time step “n”, days
ΔW_{en} – water volume influx volume during time step “n”, reservoir barrels